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### Semester Two Examination, 2020

### Question/Answer booklet

# MATHEMATICS

If required by your examination administrator, please place your student identification label in this box

**SPECIALIST**

**UNITS 1&2**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

|  |  |  |
| --- | --- | --- |
| Markers use only | | |
| Question | Maximum | Mark |
| 9 | 6 |  |
| 10 | 6 |  |
| 11 | 8 |  |
| 12 | 8 |  |
| 13 | 8 |  |
| 14 | 8 |  |
| 15 | 8 |  |
| 16 | 8 |  |
| 17 | 8 |  |
| 18 | 6 |  |
| 19 | 8 |  |
| 20 | 8 |  |
| 21 | 8 |  |
| S2 Total | 98 |  |
| S2 Wt (×0.6633) | 65% |  |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

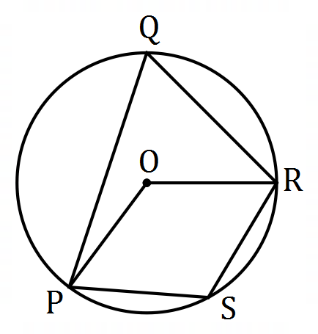
Question 9 (6 marks)

(a) Triangle has vertices and . Determine the area of this triangle after it has been transformed using the matrix . (3 marks)

(b) Show use of matrix algebra, including the coefficients of any inverse matrix used, to solve the following system of linear equations: (3 marks)

Question 10 (6 marks)

(a) Prove that the opposite angles of a cyclic quadrilateral are supplementary. (3 marks)



(b) The points and lie on  
the circle with centre so that  
 and .

Determine the size of .

(3 marks)

Question 11 (8 marks)

Two vectors are and . Determine

(a) the magnitude of . (1 mark)

(b) the angle between the directions of and . (2 marks)

(c) the value of the scalar constant so that is parallel to . (2 marks)

(d) a vector that is perpendicular to with the magnitude of . (3 marks)

Question 12 (8 marks)

The vertices of triangle are , and .

Transformation is a translation by vector .

(a) State the coordinates of the image of after triangle is transformed by . (1 mark)

Transformation is a reflection in the line .

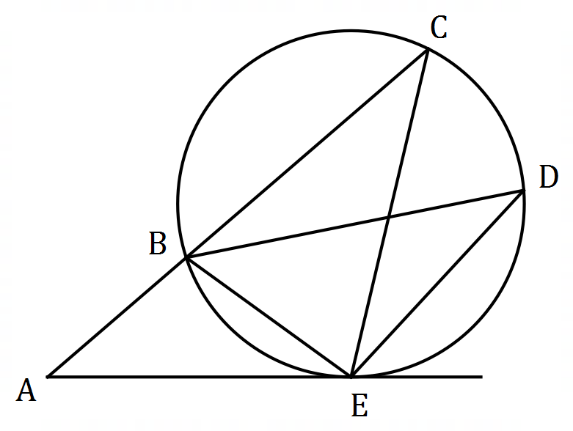
(b) Determine the transformation matrix for and state the coordinates of the image of after triangle is transformed by and then by . (3 marks)

Transformation is a rotation of clockwise about the origin.

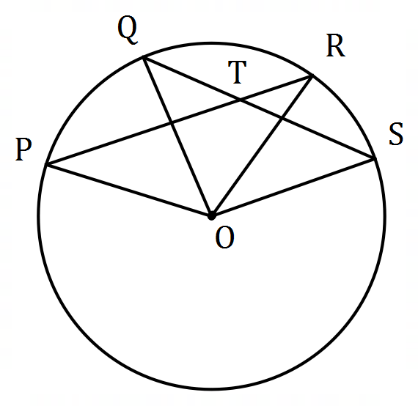
(c) Determine the exact coordinates of the image of after triangle is transformed by and then by . (3 marks)

(d) Write a matrix expression for the transformation matrix that represents the inverse of transformation followed by the inverse of transformation . There is no need to simplify your expression. (1 mark)

Question 13 (8 marks)

(a) In the diagram shown (not to scale)  
 is a straight line and and   
 lie on a circle.  
  
 is a tangent to the circle at ,   
 and .   
  
Determine, with reasons, the size  
of .

(4 marks)



(b) In the diagram shown (not to scale)  
 and lie on a circle centre   
and chords and intersect at .  
  
 and .  
  
Determine, with reasons, the size  
of .

(4 marks)

Question 14 (8 marks)

(a) State whether each of the following statements are true or false, supporting each answer with an example or counterexample.

(i) if and then . (2 marks)

(ii) , if is even then is prime. (2 marks)

(b) Prove by contradiction that is not a cyclic quadrilateral if diagonal of length cm cuts diagonal of length cm at so that cm. (4 marks)

Question 15 (8 marks)

Starting at midnight (), the temperature at a resort was observed to vary sinusoidally over the course of a day, reaching a high of C at pm after a low of C at am. Let be the time in hours from midnight.

(a) Use the above information to sketch a graph showing how varies with during the day.

(2 marks)

(b) Determine an algebraic model for as a function of . (4 marks)

(c) Use your model to determine the proportion of the day that the temperature at the resort was below C. (2 marks)

Question 16 (8 marks)

(a) Determine the number of integers between and that are

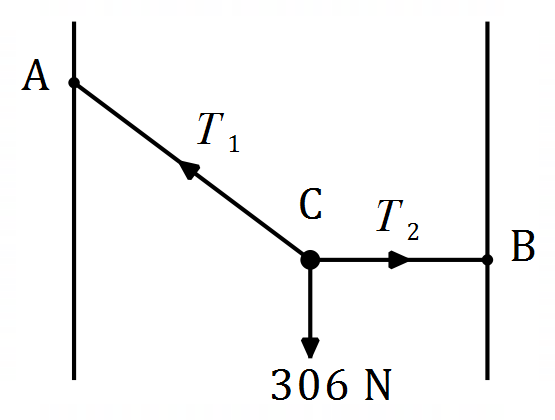
(i) divisible by . (1 mark)

(ii) divisible by or by but not by . (3 marks)

(b) A playlist offered by a music streaming service has different songs. Every time a playlist is streamed, the songs are shuffled into a random arrangement.

Show that after the playlist has been streamed times, at least of those streams began with the same songs in the same order. (4 marks)

Question 17 (8 marks)

A small object of weight N is suspended above  
level ground and between two vertical walls by two  
light inextensible strings. The walls are cm apart.

Point lies on one wall so that string is cm  
long and point lies on the other wall so that string  
 is horizontal and cm long.

(a) Determine the tension in string . (3 marks)

(b) String is lengthened so that the height of above the ground decreases by cm and . Determine the tension in string . (5 marks)

Question 18 (6 marks)

(a) Given that , determine the value(s) of the real constant so that is its own inverse. (3 marks)

(b) Let and . Determine when . (3 marks)

Question 19 (8 marks)

(a) students from Class A, from Class B and from Class C have nominated for the places available in the team for a mathematics competition. Determine the number of different teams that can be formed if

(i) the students are chosen from the same class. (2 marks)

(ii) at least students in the team are chosen from Class B. (2 marks)

(b) Prove that for , . (4 marks)

Question 20 (8 marks)

A common proof that is irrational begins by assuming that is rational, so that .

(a) Describe two properties of variables and that the proof requires, other than .

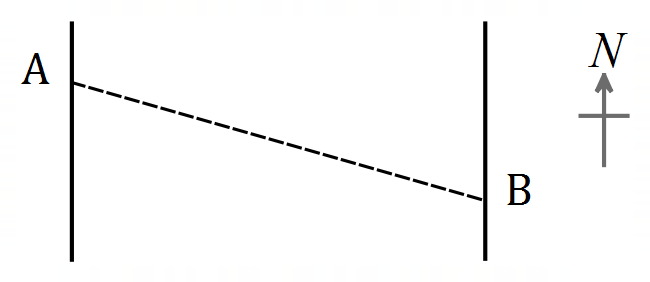
(2 marks)

The next step obtains the relationship , from which it is deduced that .

(b) Prove, using the contrapositive, that if is a multiple of then so is . (4 marks)

(c) Complete the proof that is irrational. (2 marks)

Question 21 (8 marks)

Points and lie on opposite sides of a river so that  
 is m away from on a bearing of .  
  
A uniform current flows due north in the river  
between and at m/s.

Sam can swim at a steady speed of m/s and  
plans to swim from to and then back to .

(a) Determine the bearing Sam should swim to move directly towards from . (3 marks)

(b) Show that Sam takes seconds less to swim the return leg than the first leg. (5 marks)

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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